The Combined Heat and Power Plant Cost Allocation Methods: An Overview and a New Method Based on the Linear Steam Turbine Characteristic Curve

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Dedicated to Maria Alexandrova,
Prima Ballerina of the Bolshoi Theater, People's Artist of Russia
Annotation

This paper discusses the cost allocation problem for combined heat and power generation. Firstly, I provide a review of the methods most frequently used in Russia and other countries with developed electricity and heat markets. Secondly, I develop a new thermodynamic method for cost allocation for combined production based on a linear steam turbine characteristic curve. Finally, the general principles of comparison for the observed allocation methods and their numerical results have been developed and applied and the most effective methods are identified.

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Keywords

combined heat and power production, combined heat and power plant, fuel rate, cost allocation, linear steam turbine characteristic curve

Contents

Annotation..............................................................................................................................................................2
Keywords.................................................................................................................................................................2
Preface to the English version..................................................................................................................................3
Introduction...............................................................................................................................................................4
1. A cost allocation problem statement..................................................................................................................7
2. A methods review....................................................................................................................................................9
   2.1. Classification of methods......................................................................................................................................9
   2.2. Fuel rate calculation methods used in Russia....................................................................................................10
       2.2.1. "Physical" method.........................................................................................................................................10
       2.2.2. Exergy method.............................................................................................................................................12
       2.2.3. Proportional distribution method by ORGRES..........................................................................................13
       2.2.4. Work method (Russian name is “method on under-produced electricity”)............................................16
       2.2.5. Steam parameters consideration method....................................................................................................17
   2.3. Cost allocation methods used in countries with developed electricity and heat markets.........................19
       2.3.1. Exergy method.............................................................................................................................................20
       2.3.2. Work method.............................................................................................................................................20
       2.3.3. Energy method...........................................................................................................................................20
       2.3.4. Method of an alternative way of heat supply............................................................................................21
2.3.5.Method of an alternative way of electricity supply ................................................... 22
2.3.6.Benefit distribution method .......................................................................................... 22
2.3.7.Benefit and risk-sharing method .................................................................................. 23
3.A new method of the CHP plant fuel rate calculation and cost allocation based on a steam turbine linear characteristic curve ........................................................................................................... 24
3.1.A steam turbine linear characteristic curve ..................................................................... 24
3.2.Method description ......................................................................................................... 25
3.3.Example ........................................................................................................................ 27
4.The methods comparison ................................................................................................ 28
4.1.Thermodynamic methods comparison ........................................................................... 29
4.1.1.Comparison principles ............................................................................................... 29
4.1.2.Efficiency, advantages, and disadvantages ................................................................. 29
4.1.3.Numeric analysis ....................................................................................................... 30
4.2.Economic method comparison ..................................................................................... 33
4.2.1.Comparison principles ............................................................................................... 33
4.2.2.Efficiency, advantages, and disadvantages ................................................................. 33
4.2.3.Numerical analysis ................................................................................................... 35
Conclusion ........................................................................................................................... 36
References ............................................................................................................................ 37

**Preface to the English version**

The original paper was published in the Russian language in Science and Education of Bauman MSTU journal in 2016. The present English version was published on the personal website of the author in January 2021. For reference purposes, please use **I. Chuchueva The Combined Heat and Power Plant Cost Allocation Methods: An Overview and a New Method Based on the Linear Steam Turbine Characteristic Curve, Science and Education of Bauman MSTU, 2016, №2, P. 135-165.**

The English version has been adopted for foreign readers under the following terms.

– *The wholesale electricity market of Russia terminology.*

I added a few notes that will help foreign readers to go through electricity market terminology. The basic difference between Russian and European (other developed) markets is the trading possibilities: in Russia, only day-ahead and imbalance bidding is available for participants. If a power plant doesn't participate in system balancing (imbalance phase) the deviation between the
day-ahead schedule and actual production will be traded at the imbalance price. There is neither intraday auction nor continuous trading in Russia. Such market design puts additional pressure on the day-ahead bidding procedure.

-- Fuel rate versus cost allocation problem statement.

In Russia, Combined Heat and Power (CHP) plant’s day-ahead and imbalance bidding prices are connected to their technical efficiencies by market regulation. Thus, usually, the fuel rate (fuel consumption by unit of supplied product) calculation problem is stated. In countries with developed electricity markets, the cost allocation problem is usually stated. These two problems are twin brothers: once I have a fuel rate value for a certain product, I can calculate the cost of this product. An English version is discussing the cost allocation problem meanwhile the numerical comparison is done for fuel rate values in accordance with the Russian original.

The important feature of the paper is its appendix, Chuchueva-Fuel-Rate-Eng.xlsx (xlsx-appendix). The file contains a transparent step-by-step implementation of all examined methods for a certain CHP plant operation mode. The detail of CHP plant loads and operation mode is in the sheet “CHP Operation Mode” and the fuel rate calculation and cost allocation are in the sheet “Methods.” In literature, notations of different methods are different and sometimes cumbersome. The goal of this paper and xlsx-appendix is to bring all notations for all methods to a unified background and, through that, make the comparisons easy and straightforward. Any coefficients or factors are used in methods are denoted with $\gamma_{\text{IND}}$, its index refers to the details of the coefficient or factor.

Note, some of the references are available in Russian only.

Introduction

A Combined Heat and Power (CHP) plant simultaneously produces several forms of energy: electricity, industrial steam, and heat in the form of hot water. The combined production of heat and electricity is more efficient than separate production: by 37% according to [1], by 25% according to [2], and by 30% according to [3]. The reduction of the total fuel consumption and, as a result, the total cost in combined mode production compared to separate mode is called benefit [2, 4]. Fuel cost is a variable CHP plant cost. Fixed CHP plant costs like capital, labor, etc. are not discussed in the present research; for detail of fixed cost allocation see paper [2].

The cost allocation problem for CHP plant products consists of two steps:

1) calculate the consumption of equivalent fuel per unit of each supplied product (so-called
2) calculate costs of each product based on equivalent fuel rate and price values.

The cost allocation problem has two objectives:

1) to estimate the technical efficiency of the CHP plant's operation mode

2) to increase the CHP plant’s competitive ability in both electricity and heat markets.

To achieve the first objective, several thermodynamic methods for cost allocation have been developed [2, 4]; to achieve the second objective, various economic methods have been introduced [1–3].

Under the condition of the state regulation of electricity and heat power production, the electricity and heat tariffs are strictly tied to the technical efficiency of CHP plants [2–4]. Under the power and heat market condition, the pricing is more flexible and does not have a rigid dependency on technical efficiency. On one hand, the control of the technical efficiency and reliability of a CHP plant operation is an important problem for CHP owners. On the other hand, maximum efficiency doesn't guarantee maximum profitability on the market. A CHP plant's technical efficiency together with market competitive ability and bidding strategy determine a power plant's profit in the electricity and heat markets. Sometimes, CHP plant market activities might affect market price [1–3].

In Russia, despite the ten-year period of the Wholesale Electricity and Capacity Market development since 2006, CHP plants market bidding prices are still tightly connected to their technical efficiency by market regulation [5]. Moreover, in Russia, the least correct thermodynamic method — the so-called physical method — is officially applied to determine CHP plant technical efficiency [3, 4]. At the same time, neither of the economic methods for cost allocation is being applied for the development of market bidding strategies [5].

The objectives of this work are as follows:

a) A review of the cost allocation methods that are most frequently applied in Russia, as well as in other countries with developed electricity and heat markets.

b) Development of a new thermodynamic cost allocation method based on the linear steam turbine characteristic curve.

c) Comparison of the efficiencies within the thermodynamic and economic method groups based on the specified principles and numerical results.
In the review part of the work, the five thermodynamic methods that are most frequently applied are considered, namely: physical method, exergy method, proportional distribution method by ORGRES\(^1\), work method, and steam parameters consideration method [1–4]. Then, the five most commonly applied economic methods are examined: energy method, method of an alternative way of heat supply, method of an alternative way of electricity supply, benefit distribution method, and benefit and risk-sharing method [1, 2, 6, 7].

A new method for cost allocation based on the linear steam turbine characteristic curve is proposed in this paper. This method has two advantages. Firstly, it allows us to consider the differences between various forms of energy, for example, steams with different parameters in CHP plant products [4, 8]. Secondly, it is simple and requires a small amount of input data.

The principles for comparing the efficiencies of each of two groups of methods, thermodynamic and economic, are specified. For a certain steam turbine operation mode, both fuel rates and costs are calculated using each of the mentioned methods. The comparison of the efficiencies of the thermodynamic and economic groups proves that the most effective thermodynamic methods are the exergy method and the work method. The most efficient economic methods are the benefit and risk-sharing method and the method of an alternative way of heat supply.

An important result of the study is the conclusion that the physical method, i.e. the official method for cost allocation adopted in Russia, is the least effective thermodynamic method. The widespread application of this method and its rigid regulatory connection with the day-ahead and imbalance bidding prices impede the electricity and heat market development in the country.

The work has the following structure. The first section contains the problem statement for both the fuel rate calculation and cost allocation. The second section contains a review of the methods. The review consists of two parts: in the first part, I examine the methods applied in Russia; in the second, I examine the methods applied in other countries with developed electricity and heat markets. The third section introduces the new cost allocation method based on the linear steam turbine characteristic curve. In the fourth section, I develop the principles for comparing the efficiencies of the thermodynamic and economic methods and order the methods from most to least efficient within the group. Finally, the numerical results of the methods are analyzed.

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\(^1\) ORGRES is the name of a Russian company [http://www.orgres-f.ru](http://www.orgres-f.ru)
1. A cost allocation problem statement

A CHP plant burns fuel to supply several energy products that are, in most cases, electricity, industrial steam, and heat. A fraction of supplied products is consumed by the CHP plant for its own needs. These are, so-called, auxiliary services. The CHP plant net production is the gross supplied products minus the auxiliary services [4, 5]. Let us take a closer look at each CHP plant product.

- **Electricity** $E$. The amount of electricity generated by a CHP plant is determined by the market mechanism based on the day-ahead and intraday bid a CHP plant has made [5]. In accordance with the wholesale electricity market of Russia's rules, the CHP plant day-ahead and intraday bidding prices should be estimated based on the equivalent fuel rate value for one MWh supply [5, 9]. In other words, the power exchange controls the CHP plant bidding prices and is allowed to request a technical foundation for “weird” values (usually too high). The money income for the generated electricity is regulated by the wholesale electricity market rules [5]. The volume of electricity for its own usage, denoted as $E^{AUX}$, is being bought on the market [5].

- **Industrial steam** $Q_M$. The bottom index $M$ refers to medium-pressure steam. As a rule, a medium-pressure steam of 6–35 atm and a temperature around 150–250°C is consumed by industrial enterprises for technological needs. That's why it's usually referred to as industrial steam. A steam supply schedule is defined in the contract between a CHP plant and an enterprise and is mandatory for a CHP plant. The payment for the supplied steam is made under the terms of the contract. The volume of the CHP plant’s own usage of industrial steam is denoted as $Q^{AUX}_M$.

- **Heat** in the form of hot water $Q_L$. The bottom index $L$ refers to low-pressure steam that heats up the water. The water of a temperature 50–120°C is consumed by households and enterprises. The water heating schedule is calculated using the outside temperature special dependency [10]. The lower is the outside temperature, the higher is the CHP plant's heat load. Similar to industrial steam, the heat production schedule is mandatory for CHP plants. In Russia, the money income for the supplied heat is based on an average monthly equivalent fuel rate for one Gcal of supply. The tariff is approved by the Federal Tariff Service for a given year [11]. The heat

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2 In Russia, the intraday market is the pure balancing market where power plants offer their change of power production to the System Operator to balance supply and demand. Neither GE/GB style intraday auction nor continuous trading phase are applied in the country. The deviation between the day-ahead schedule and actual production and consumption is being traded directly by the imbalance price.

3 In Europe, a net position is being traded. Completely opposite, in Russia, all power plants including CHP plants sell their gross supply and buy their auxiliary services supply at the same price. These two trades provide transparency of power plant operation.
trade in other countries is observed in [2]. The volume of the CHP plant's own usage of hot water is denoted as $Q^\text{AUX}_L$.

To generate the products, CHP plants burn different types of fuels: natural gas, domain gas, fuel oil, coal, biomass, etc. [1]. In Russia, in order to simplify the fuel rate calculation, the consumption of various types of fuel is reduced to the consumption of, so-called, equivalent fuel [4]. The calorific value of equivalent fuel is constant and equal to 7,000 kcal/kg [12]. On paper, the equivalent fuel price is equal to 100 EUR/tef (ton of equivalent fuel)$^4$.

As we may see from above, for Russia, two out of three product costs are based on the fuel rate value. There is a price negotiation between counterparts only with industrial steam supply. This is the reason for splitting the cost allocation problem in this paper into two steps: first, the fuel rate is calculated, second, cost allocation is completed. To harmonize numeric estimations, for economic methods the same approach is implemented. In other words, for all the methods, firstly, the technical side of the problem is solved, and secondly, the economical one.

Three products $E$, $Q^\text{M}$, $Q^\text{L}$ generation could be considered within different time intervals. The interval depends on the purposes of the calculation. In Russia, for electricity cost allocation, an interval of one hour is used; to calculate the steam and heat cost allocation an interval of one month is used.

Note that the CHP plant's own usage, auxiliary services, of electricity $E^\text{AUX}$ is a part of the gross electricity generation $E$; in the same manner, the industrial steam auxiliary services $Q^\text{AUX}_M$ is a part of the gross production of industrial steam $Q^\text{M}$; the hot-water auxiliary services $Q^\text{AUX}_L$ is a part of the gross production of hot water $Q^\text{L}$.

**Step 1. Fuel rate calculation**

In the first step, the fuel rate calculation problem is solved. The fuel rate calculation comprises the following simple equations:

- The electricity fuel rate

$$b_E = \frac{B_E}{E - E^\text{AUX}_E}; \quad (1)$$

- The industrial steam fuel rate

$^4$ In the Russian version of the paper, 5,000 RUB/tef is used.
\[ b_M = \frac{B_M}{Q_M - Q_{M AUX}} \]  

-- The heat fuel rate

\[ b_L = \frac{B_L}{Q_L - Q_{L AUX}} \]  

Where \( B_E, B_M, B_L \) are the parts of the total consumption of equivalent fuel \( B \) needed for the generation of electricity, industrial steam, and heat, respectively. The values of \( B_E, B_M, B_L \) should meet the fuel balance:

\[ B = B_E + B_M + B_L \]  

The main input data for the calculation are:

- the values of the CHP plant's loads \( E, E_{AUX}, Q_M, Q_{M AUX}, Q_L, Q_{L AUX} \)
- the total consumption of equivalent fuel \( B \).

The scope of additional input data depends on the applied method (section 2).

MWh is used as a unit for the values of the CHP plant loads; tef (tons of equivalent fuel) – for the consumption of equivalent fuel. Thus, the fuel rates \( b_E, b_M, b_L \) are measured in tef/MWh; if necessary, values can be converted into kgeff/MWh, gfe/kWh, gfe/GJ, kgeff/Gcal, EUR/MWh, EUR/kWh, and several other units [4, 9].

**Step 2. A cost allocation**

In the second step, the supplied product cost per unit is obtained by multiplying equivalent fuel rate values \( b_E, b_M, b_T \), tef/MWh, and equivalent fuel price, EUR/tef. This step is performed in the xlsx-appendix to the paper. Numerical analysis of the methods is done based on fuel rate values in tef/MWh. The pretext is given in the Preface to the English version of this paper.
2. A methods review

2.1. Classification of methods

The cost allocation methods are usually divided into two groups [1–3, 6]:

• thermodynamic
• economic.

The purpose of thermodynamic methods is to determine the technical efficiency of the CHP plant operation modes. The methods consider the thermodynamic details of the combined heat and power production technology [4]. These methods include:

- “physical” (the other names in Russian literature are heat, balance, and enthalpy) (section 2.2.1)
- exergy (sections 2.2.2, 2.3.1)
- proportional distribution by ORGRES (section 2.2.3)
- work (sections 2.2.4, 2.3.2)
- steam parameters consideration (section 2.2.5).

The purpose of economic cost allocation methods is to increase the CHP plant product competitiveness in the electricity and heat markets [2, 3, 6]. The CHP plant competitive ability in the markets is determined by the cost of its products: the price of the product to be supplied should be lower than the marginal market price [5]. As mentioned above, the cost value of the CHP plant product is an economic indicator based on the fuel rate value. The list of these methods includes:

- energy (section 2.3.3)
- an alternative way of heat supply (section 2.3.4)
- an alternative way of electricity supply (section 2.3.5)
- benefit distribution (section 2.3.6)
- benefit and risk-sharing (section 2.3.7).

2.2. Fuel rate calculation methods used in Russia

Despite the Russian wholesale electricity market development, the problem of fuel rate
calculation is solved exclusively by thermodynamic methods [3, 4, 9, 11]. According to the market rules, the result of the fuel rate calculation is used to determine the day-ahead and intraday bidding price for one MWh of electricity supply [5]. In the case of a “weird” participant bidding price, the exchange is allowed to demand the details of fuel rate calculation to evaluate the participant price. Speaking about Russian methods, I use the fuel rate term.

2.2.1. “Physical” method

The “physical” method for fuel rate calculation for CHP plant production of electricity, industrial steam, and heat was the official one in Russia until 1996 [3]. Since 2013, this method has become official in Russia again [9]. Sometimes, this method is called the heat [9], balance [4], or enthalpy [8] method. Ironically, the method doesn’t reflect the physics of the combined heat and power production process, thus, I put its name in the quotes.

The “physical” method requires an additional input value: the total high-pressure steam consumption $Q_0$, MWh.

According to the “physical” method, the fuel rate calculation is carried out in two steps.

1) Distribute the total consumption of equivalent fuel between different CHP plant products

The consumption of equivalent fuel for electricity generation $Q_E$ is given by the expression

$$B_E = \frac{Q_E}{Q_0} \cdot B,$$

where $Q_E$ is the high-pressure steam consumption for electricity generation, MWh. The value of $Q_E$ is calculated from the steam balance:

$$Q_0 = Q_E + Q_M + Q_L. \quad (5)$$

The consumption of equivalent fuel for industrial steam and heat production are given by the corresponding formulas:

$$B_M = \frac{Q_M}{Q_0} \cdot B,$$

$$B_L = \frac{Q_L}{Q_0} \cdot B.$$
2) Calculate fuel rate

The fuel rates \( b_E, b_M, b_L \) are calculated using equations (1) – (3). In case of gross generation fuel rates, auxiliary services \( E^{AUX}, Q_M^{AUX}, Q_L^{AUX} \) should be set to zero MWh; for the net generation fuel rates the auxiliary services values should be defined for a certain CHP plant operation mode. In the xlsx-appendix, \( E^{AUX} = 3 \) MWh and \( Q_M^{AUX} = Q_L^{AUX} = 0 \) MWh. This paragraph is being applied for all final steps of the following methods.

2.2.2. Exergy method

The exergy method is considered the most “fair” and correct thermodynamic method [2, 4, 6]. The concept of exergy is to evaluate the quality of different forms of energy [13]. The method is adopted in countries with developed electricity markets to assess the technical efficiency of CHP plants’ operation modes [2, 6] (section 2.3.1).

The method requires additional input parameters describing the CHP plant operation mode:

– enthalpy and entropy of various types of steam and their condensate
– outside temperature.

The exergy method for each CHP plant product consists of three steps.

1) Find the total energy of the CHP plant products

Total CHP plant operation mode exergy \( E_{TOTAL} \) in MWh is calculated using the formula:

\[
E_{TOTAL} = E_E + E_M + E_L.
\]

The values of exergies \( E_E, E_M, E_L \), MWh are calculated using the following expressions:

– Electricity exergy

\[
E_E = E
\]

– Industrial steam exergy

\[
E_M = Q_M \left(1 - \frac{T_{env}}{T_M}\right)
\]

5 Be careful, I use \( E \) for the CHP plant electricity production; and here, \( E_E \) means exergy for electricity product and total exergy is denoted with \( E_{TOTAL} \). You may ask: why do I do this? Simply because in English language papers, the most often used notation for electricity – \( E \), for exergy – \( E \) either.

12
− Heat exergy

\[ E_L = Q_L \left(1 - \frac{T_{\text{env}}}{T_L}\right). \]

Here, \( T_{\text{env}}, \) K, is the outside temperature; \( T_M, T_L, \) K, are average industrial steam temperature \( Q_M \) and average low-pressure steam temperature required for heat \( Q_L \) production.

The value of \( T_M \) is defined by the following formula:

\[ T_M = \frac{H_M^c - H_M}{S_M^c - S_M}, \]

where \( H_M, S_M \) are enthalpy, kJ/kg, and entropy, kJ/kg·K, of industrial steam; \( H_M^c, S_M^c \) are enthalpy and entropy of industrial steam condensate correspondingly.

Analogously, \( T_L \) value is derived from the expression:

\[ T_L = \frac{H_L^c - H_L}{S_L^c - S_L}, \]

where \( H_T, S_T \) are enthalpy and entropy of low-pressure steam which is required for heat production \( Q_L; H_L^c, S_L^c \) are enthalpy and entropy of low-pressure steam condensate.

2) Distribute of the total exergy by CHP plant products

The equivalent fuel consumption for electricity production is determined by the expression:

\[ B_E = \frac{E_E}{E_{\text{TOTAL}}} \cdot B. \]

Correspondingly, the equivalent fuel consumption for the industrial steam and heat production equal:

\[ B_M = \frac{E_M}{E_{\text{TOTAL}}} \cdot B, \]

\[ B_L = \frac{E_L}{E_{\text{TOTAL}}} \cdot B. \]

3) Calculate fuel rate
The fuel rates \( b_E, b_M, b_L \) are calculated using equations (1) – (3).

### 2.2.3. Proportional distribution method by ORGRES

This method used to be an official method for preparing reports on the CHP plants' technical efficiency within the period 1996 – 2013 [4, 14].

This method requires the following additional input data:

– the total consumption of high-pressure steam
– enthalpies of various types of steam
– enthalpy of the high-pressure steam condensate
– factor \( k \) for regenerative heating of feed water.

The method consists of four steps.

1) **Find steam coefficients**

The industrial steam coefficient is calculated using the formula:

\[
\gamma_M = \frac{H_M - H_0^c}{H_0 - H_0^c} \left( 1 + k \left( \frac{H_0 - H_M}{H_0 - H_0^c} \right) \right),
\]

where \( H_0 \) and \( H_0^c \) are enthalpies and entropies of high-pressure steam and its condensate, kJ/kg.

The value of factor \( k \) depends on CHP plant equipment nomenclature and is provided by ORGRES [14].

Analogously, the value coefficient of low-pressure steam is determined as:

\[
\gamma_L = \frac{H_L - H_0^c}{H_0 - H_0^c} \left( 1 + k \left( \frac{H_0 - H_L}{H_0 - H_0^c} \right) \right).
\]

2) **Calculate electricity fuel rate**

The fuel rate for total electricity production \( E \) is calculated by the formula:

\[
B_E = B \cdot \gamma_{dE},
\]

where \( \gamma_{dE} \) is a special coefficient for electricity production. This coefficient is derived using the expression:

\[\text{This coefficient notation is the only difference from } \gamma, \text{ for details see the xlsx-appendix.}\]
\[ \gamma_{dE} = \frac{Q_E + \Delta Q_E}{Q_E + \Delta Q_E + Q_M + Q_L}. \]

Here, \(Q_E\) is the high-pressure steam consumption for electricity production that was obtained in the “physical” method (5); \(\Delta Q_E\) is additional high-pressure steam consumption for electricity production, MWh equals:

\[ \Delta Q_E = Q_M \cdot (1 - \gamma_M) + Q_L \cdot (1 - \gamma_L). \]

At the end of this step, the electricity fuel rate \(b_E\) is calculated with equation (1).

3) **Calculate industrial steam fuel rate**

The industrial steam fuel rate is determined by the statement:

\[ b_{M}^{\text{total}} = b_{ML} - \Delta b_M, \]

where \(b_{ML}\) is the total fuel rate for industrial steam and heat production combined, tef/MWh; \(\Delta b_M\) is a reduction of industrial steam fuel rate, tef/MWh.

The total fuel rate for industrial steam and heat production combined is calculated as follows:

\[ b_{ML} = \frac{B_{ML}}{Q_M + Q_L}. \]

Here, \(B_{ML}\) is the total fuel consumption for industrial steam and heat production combined determined as the difference:

\[ B_{ML} = B - B_E. \]

The value of \(B_E\) is calculated on the previous step (8).

The reduction for industrial steam fuel rate is obtained with:

\[ \Delta b_M = \Delta b_{ML} \cdot \frac{(1 - \gamma_M)}{(1 - \gamma_{ML})}. \]

Here, \(\Delta b_{ML}\) is the total reduction of the total fuel rate for industrial steam and heat production combined, tef/MWh; \(\gamma_{ML}\) is an average of the steam coefficient for industrial steam and low-pressure steam. The value \(\Delta b_{ML}\) is calculated by the formula:
\[ \Delta b_{ML} = b_{ML} \left( \frac{Q_E + \Delta Q_E + Q_M + Q_L}{Q_E + Q_M + Q_L} - 1 \right) . \] (10)

The value \( \gamma_{ML} \) is determined by the statement:

\[ \gamma_{ML} = \frac{Q_M \cdot \gamma_M + Q_L \cdot \gamma_L}{Q_M + Q_L} . \] (11)

The industrial steam net production fuel rate is calculated as follows:

\[ b_M = b_{total} \cdot \frac{Q_M}{Q_M - Q_{M AUX}} . \]

In case \( Q_{M AUX} = 0 \), the fuel rate for both total and net production are equal to \( b_{total} \).

4) Calculate heat fuel rate

Analogously with industrial steam, the total heat production fuel rate is defined with the expression:

\[ b_{L total} = b_{L} - \Delta b_{L} . \]

Here, \( \Delta b_{L} \) is the reduction of industrial steam and heat fuel rate combined, tef/MWh, it's equal to:

\[ \Delta b_{L} = \Delta b_{ML} \cdot \frac{(1 - \gamma_L)}{(1 - \gamma_{ML})} . \]

Values \( \Delta b_{ML}, \gamma_{ML} \) were calculated with the formulas (10) and (11) above.

Thus, the net heat production fuel rate equals:

\[ b_{L} = b_{L total} \cdot \frac{Q_L}{Q_L - Q_{L AUX}} . \]

2.2.4. Work method (Russian name is “method on under-produced electricity”)

This method is based on taking into account the reduction of electricity production due to applying high-pressure steam for industrial steam and heat production [2, 4]. The method is applied both in Russia and in other countries with developed electricity and heat markets.

The method requires the input data about additional CHP plant operation modes as presented in Table 1.

Table 1: Input data for additional CHP plant operation modes
Electricity productions $E^1$, $E^2$, $E^3$ are determined in accordance with turbine characteristic curves [15] for the quantities of high-pressure steam, industrial steam, and heat production indicated in the table.

The method consists of the following three steps.

1) **Calculate electricity fuel rate**

Fuel consumption for electricity production is defined with the expression:

$$B_E = B \cdot \frac{E}{E^3}.$$  
Then, the electricity fuel rate is calculated with formula (1).

2) **Calculate industrial steam fuel rate**

The fuel consumption for industrial steam production is calculated as follows:

$$B_M = \frac{B_{ML}}{1 + \frac{1}{\gamma_{ML}}}.$$  
In this method, coefficient $\gamma_{ML}$ defines the industrial steam to heat ratio. The value of the ratio is defined by the expression:

$$\gamma_{ML} = \frac{E^3 - E^1}{E^3 - E^2}.$$  
The value of $B_{ML}$ is calculated with formula (9); the industrial steam fuel rate with formula (2).

3) **Calculate heat fuel rate**

The fuel consumption for heat production comes from:
The value of $\gamma_{ML}$ is calculated in the previous step.

The heat fuel rate is calculated in accordance with formula (3).

2.2.5. **Steam parameters consideration method**

This method is based on taking into account the quality of different kinds of steam using the steam quality coefficients calculated in accordance with the proportional distribution method by ORGRES (section 2.2.3).

Additional input data:

- total high-pressure steam consumption
- the enthalpies of the steams
- the enthalpy of high-pressure steam condensate
- factor $k$ for regenerative heating of feed water.

The method consists of three steps.

1) **Calculate steam coefficients**

Here, the steam coefficients $\gamma_M$, $\gamma_L$ are calculated by the formulas (6), and (7) respectively.

2) **Calculate electricity fuel rate**

The fuel consumption for electricity production is defined with the expression:

$$B_E = B \cdot \gamma_{BE},$$

(12)

where $\gamma_{BE}$ is a new coefficient which is obtained as follows:

$$\gamma_{BE} = \frac{Q_0 - (Q_M \gamma_M + Q_L \gamma_L)}{Q_0}. $$

The electricity fuel rate is calculated in accordance with formula (1).

3) **Calculate industrial steam fuel rate**

Analogously to (12), the fuel consumption for industrial steam production equals:
\[ B_M = B \cdot \gamma_{B_M}, \]

where \( \gamma_{B_M} \) is another additional coefficient determined as follows:

\[ \gamma_{B_M} = \frac{Q_M \cdot \gamma_M}{(Q_M \cdot \gamma_M + Q_L \cdot \gamma_L)}. \]

The industrial steam fuel rate is calculated in accordance with formula (2).

4) Calculate heat fuel rate

As previously, the fuel consumption for heat production equals:

\[ B_L = B \cdot \gamma_{B_L}, \]

Where coefficient \( \gamma_{B_L} \) is calculated by the expression:

\[ \gamma_{B_L} = \frac{Q_L \cdot \gamma_L}{(Q_M \cdot \gamma_M + Q_L \cdot \gamma_L)}. \]

The heat fuel rate is defined by formula (3).

2.3. Cost allocation methods used in countries with developed electricity and heat markets

In countries with developed electricity and heat markets, the cost allocation problems are discussed [1, 2, 6]. The problem is solved in order is to increase the competitiveness of CHP plants in the electricity and heat markets. The result of the calculation is the production cost per unit for CHP plant products (electricity, industrial steam, heat).

In this paper, Western methods for estimating the production cost per unit for a CHP plant are slightly adapted and split into two steps:

1) fuel rate calculation, tef/MWh

2) product cost calculation based on obtained fuel rate and equivalent fuel cost, EUR/MWh (see details below).

This approach allows us to compare fuel rate values for Russian and Western methods combined. Thus, the following agreements are adopted.

− Total fuel cost:

\[ C_B = B \cdot Z_B, \]

where \( C_B \) is fuel cost, EUR; \( Z_B \) is the equivalent fuel price, EUR/tef.
− Product costs:

\[
Z_E = b_E \cdot Z_B ; \quad (13)
\]

\[
Z_M = b_M \cdot Z_B ; \quad (14)
\]

\[
Z_L = b_L \cdot Z_B . \quad (15)
\]

Here, \( Z_E \), \( Z_M \), \( Z_L \) are CHP plant product costs, EUR/MWh. In Russia, these values are commonly referred to as *product fuel cost* [5].

Since the costs of industrial steam production are not accounted for in some of the works [2, 6], the methods considered in these works have been supplemented based on the following considerations:

− An alternative to the combined production of industrial steam is the steam generated by a reduction-cooling unit

− Industrial steam is supplied at a contract price that is not subjected to market risks.

2.3.1. Exergy method

The exergy method used in countries with developed electricity and heat markets is identical to the exergy method used in Russia (section 2.2.2). The method is mainly applied to assess the technical efficiency of a CHP plant operation.

2.3.2. Work method

The work method is identical to the method of under-produced electricity used in Russia (section 2.2.4). The method is mainly applied to assess the technical efficiency of a CHP plant's operation.

2.3.3. Energy method

According to this method, the cost allocation problem is solved in the following two steps [2].

1) Allocate total fuel consumption by product

The fuel consumption for electricity production equals:

\[
B_E = B \cdot \frac{E}{E + Q_M + Q_L} .
\]
Similarly, the fuel consumption for industrial steam and heat productions are calculated as follows:

\[ B_M = B \cdot \frac{Q_M}{E + Q_M + Q_L}, \]

\[ B_L = B \cdot \frac{Q_L}{E + Q_M + Q_L}. \]

2) Calculate fuel rates and product costs

The specific fuel rates are calculated based on formulas (1) – (3); product costs on formulas (13) – (15).

2.3.4. Method of an alternative way of heat supply

This method assumes that an alternative to the combined heat production is the production of heat \( Q_L \) by a hot-water boiler and industrial steam \( Q_M \) by a reduction-cooling unit [1, 2].

Additional input data:

- hot-water boiler efficiency
- steam boiler efficiency
- reduction-cooling unit efficiency is taken as 100%.

The cost allocation problem is solved in two steps.

1) Allocate total fuel consumption by product

The consumption of equivalent fuel for alternative heat production is given by the statement:

\[ B_L = B_L^a = \frac{Q_L}{\eta_{WB}} \cdot \gamma_{\text{tef}}, \quad (16) \]

where \( \gamma_{\text{tef}} \) is a constant equal to 0.123 tef/MWh.

The fuel consumption for industrial steam production is calculated analogously:

\[ B_M = B_M^a = \frac{Q_M}{\eta_{SB}} \cdot \gamma_{\text{tef}}, \quad (17) \]

Here, \( \eta_{WB} \) is a water-boiler efficiency, \( \eta_{SB} \) is a steam boiler efficiency.

Note, that a water-boiler efficiency is not a technical characteristic of a certain boiler but a
variable that may take a value in the range from 92% to 112%. The variable allows flexibility in the product's cost allocation depending on the market price expectations. Recommendations for setting this value are given in the work [2].

The fuel consumption for electricity production is given by the expression:

\[ B_E = B - (B_M + B_L) \].

**2) Calculate fuel rates and product costs**

The specific fuel rates are calculated based on formulas (1) – (3); product costs on formulas (13) – (15).

**2.3.5. Method of an alternative way of electricity supply**

This method is based on the assumption that the condensational electricity supply is an alternative to the electricity co-generation [1, 2]. A CHP plant condensational operation mode is the mode when CHP plant supplies electricity only, the output of the other products is equal to 0.

An additional input value is the consumption of high-pressure steam \( Q_0^c \) for the power production \( E \) in condensational operation mode. The value of \( Q_0^c \) is determined in accordance with the turbine characteristic curves [15].

The method consists of two steps.

**1) Allocate total fuel consumption by product**

The fuel consumption for electricity production equals:

\[ B_N = B^a_N = B \cdot \frac{Q_0^c}{Q_0} \].

(18)

Analogously, fuel consumption for industrial steam and heat production are derived according to:

\[ B_M = (B - B_E) \cdot \frac{Q_M}{Q_M + Q_L} \],

\[ B_L = (B - B_E) \cdot \frac{Q_L}{Q_M + Q_L} \].

**2) Calculate fuel rates and product costs**
The specific fuel rates are calculated based on formulas (1) – (3); product costs on formulas (13) – (15).

2.3.6. Benefit distribution method

This method is based on the allocation of the consumption of equivalent fuel in proportion to the consumption of alternative production of all CHP plant products. The considered method is a combination of the method of alternative heat supply (section 2.3.4) and the method of alternative electricity supply (section 2.3.5) [2].

The additional input data are:

− hot-water boiler efficiency
− steam boiler efficiency
− reduction-cooling unit efficiency is taken as 100%
− the consumption of high-pressure steam $Q_0^c$ for the power production $E$ in condensational mode.

The method consists of two steps.

1) Allocate total fuel consumption by product

The specific fuel consumption for the production of electricity, industrial steam, and heat are calculated by formulas (18), (17), and (16) respectively.

Note that, in this method, the real technical water-boiler efficiency is used in contrast to the method of alternative heat supply, where this value is a variable [2, 6].

Total alternative fuel consumption for three product combined equals:

$$B^a = B^a_E + B^a_M + B^a_L.$$ 

The fuel consumption for electricity production is calculated by the expression:

$$B_N = B \frac{B^a_N}{B^a}.$$ 

Analogously, fuel consumption for industrial steam and heat production are derived as follows:
\[ B_M = B \cdot \frac{B_M^a}{B^a} , \]
\[ B_L = B \cdot \frac{B_L^a}{B^a} . \]

2) **Calculate fuel rates and product costs**

The specific fuel rates are calculated based on formulas (1) – (3); product costs on formulas (13) – (15).

2.3.7. **Benefit and risk-sharing method**

This method allows taking into account the risk of the market electricity price changes [2]. For this method, an additional input value is the expected change in the market electricity price \( r \in [-1; 1] \) that reflects the market expectation. The value of \( r \) is positive when an increase in electricity prices is expected, and negative otherwise.

In this work, the risk \( r \) is applied at fuel rate calculation. Remember, that fuel rates are connected to product costs by expressions (13) – (15).

According to this method, the problem is solved in the following two steps.

1) **Calculate fuel rates and product costs**

The calculation is done in accordance with the alternative heat supply method (section 2.3.4).

2) **Adjust fuel rates**

To increase competitiveness in the electricity and heat markets simultaneously, the consumption of equivalent fuel is redistributed between these two market products. The main product is electricity, as its market price is the most volatile. The value of \( r \) is used to re-evaluate the fuel rate as follows:

\[ b^\text{market}_E = b_E^a (1 + r) . \]

To maintain the fuel balance (4) the heat fuel rate is adjusted the following way:

\[ b^\text{market}_L = \frac{(B - B_M^a) - b^\text{market}_E (E - E^\text{AUX})}{Q_L - Q^\text{AUX}_L} . \]

Therefore, with an increase in the electricity fuel rate, the heat fuel rate is reduced, and vice
versa. The values of $b_{E}^{\text{market}}$ and $b_{L}^{\text{market}}$ are the basis for market bidding. The value of $b_{M}$ remains constant.

3. A new method of the CHP plant fuel rate calculation and cost allocation based on a steam turbine linear characteristic curve

3.1. A steam turbine linear characteristic curve

A linear characteristic curve has been developed for both gas and steam turbines during the CHP plant optimization project. The details of the project are discussed in my previous paper [16]. Currently, it's in Russian, but I've scheduled its translation to English in 2021. In the present paper, a steam turbine is considered only.

A linear characteristic curve for steam turbine $g$ looks as follows [16]:

$$Q_{0}^{g}=\alpha_{E}^{g}E_{E}^{g}+\alpha_{M}^{g}Q_{M}^{g}+\alpha_{L}^{g}Q_{L}^{g}+\alpha_{0}^{g}+\epsilon_{g}, \quad (19)$$

where $g \in [1:G]$ is a steam turbine number; $\alpha_{i}^{g}$ are linear coefficients; $\epsilon_{g}$ is the error of linearization. The algorithm to get a linear characteristic curve and $\alpha_{i}^{g}$ values for both steam and gas turbines is developed in work [16]. An average mean absolute percentage error of linearization of various types of turbines comprises 0.6%. See error details in the paper [16].

The total consumption of high-pressure steam by the turbine $g$ is split into three parts:

$$Q_{0}^{g}=Q_{0}^{g}(E)+Q_{0}^{g}(Q_{M})+Q_{0}^{g}(Q_{L}). \quad (20)$$

Here, $Q_{0}^{g}(E)$, $Q_{0}^{g}(Q_{M})$, $Q_{0}^{g}(Q_{L})$ are consumption of high-pressure steam for production electricity, industrial steam, and heat, respectively, MWh.

Define the consumption of high-pressure steam for the production of electricity based on a linear characteristic curve (19):

$$Q_{0}^{g}(E)=\alpha_{E}^{g}E_{E}^{g}+\alpha_{0}^{g}. \quad (21)$$

Value $\alpha_{0}^{g}$ is called idle consumption, i.e. consumption of high-pressure steam when the turbine doesn’t generate any output. This value is allocated for electricity generation [17].

Consumption of high-pressure steam for industrial steam and heat production is determined by the following statements:
\[ Q_0^g(Q_M^g) = \alpha_M^g \cdot Q_M^g, \]
\[ Q_0^g(Q_L^g) = \alpha_L^g \cdot Q_L^g. \]

3.2. Method description

This method is a thermodynamic method for the fuel rate calculation. The method is based on the high-pressure steam equation (20).

The additional input data are the coefficients of the linear characteristic curve for the steam turbine of a CHP plant [16].

According to this method, the problem is solved in the following three steps.

1) Find high-pressure steam consumption for CHP plant products

For a set of CHP plant steam turbines, the consumption of high-pressure steam for electricity generation is:

\[ Q_0(E) = \sum_{g=1}^{G} Q_0^g(E^g) = \sum_{g=1}^{G} \left( \alpha_E^g \cdot E^g + \alpha_0^g \right), \]

where \( Q_0(E) \) is the total high-pressure steam consumption for CHP plant electricity production \( E \). Note, that a similar technique has been developed for gas turbines\(^7\).

In the same style, I define equations for total high-pressure steam consumption for industrial steam and heat production:

\[ Q_0(Q_M) = \sum_{g=1}^{G} Q_0^g(Q_M^g) = \sum_{g=1}^{G} \alpha_M^g \cdot Q_M^g; \]
\[ Q_0(Q_L) = \sum_{g=1}^{G} Q_0^g(Q_L^g) = \sum_{g=1}^{G} \alpha_L^g \cdot Q_L^g. \]

2) Allocate total fuel consumption by CHP plant products

The fuel consumption for electricity production equals:

\(^7\) If you’re interested in equations for the gas turbine, please, contact me.
\[ B_E = B \cdot \frac{Q_0(E)}{Q_0} . \]

Analogously, the fuel consumption for industrial steam and heat production is calculated according to:

\[ B_M = B \cdot \frac{Q_0(Q_M)}{Q_0}, \]

\[ B_L = B \cdot \frac{Q_0(Q_L)}{Q_0}. \]

3) Calculate fuel rates and product costs

The specific fuel rates are calculated based on formulas (1) – (3); product costs on formulas (13) – (15).

3.3. Example

For the CHP plant under consideration in xlsx-appendix, the co-generation is carried out by a steam turbine ST-135. A diagram of the CHP plant operation mode is given in [4].

Input data:

- Electricity production \( E = 80 \) MWh , electricity auxiliary service \( E^{AUX} = 3 \) MWh
- Industrial steam production \( Q_M = 144.2 \) MWh , industrial steam auxiliary service \( Q_M^{AUX} = 0 \) MWh
- Heat production \( Q_L = 93 \) MWh , heat auxiliary service \( Q_L^{AUX} = 0 \) MWh
- Total fuel consumption \( B = 48.4 \) tef
- Linear coefficients for ST-135 \( \alpha_E = 1.8492 \), \( \alpha_M = 0.7146 \), \( \alpha_L = 0.2820 \), \( \alpha_0 = 95.6873^8 \).

1) Find high-pressure steam consumption for CHP plant products

High-pressure steam consumption for electricity production equals:

\[ Q_E^{AUX} = \]
\[ Q_0(E) = \alpha_E \cdot E + \alpha_0 = 1.8492 \cdot 80 + 95.6873 = 243.6 \text{ MWh} \ . \]

High-pressure steam consumption for industrial steam and heat production:

\[ Q_0(Q_M) = \alpha_M \cdot Q_M = 0.7146 \cdot 144.2 = 103.1 \text{ MWh} \ , \]
\[ Q_0(Q_L) = \alpha_L \cdot Q_L = 0.2820 \cdot 93 = 26.2 \text{ MWh} \ . \]

Total high-pressure steam consumption is the sum of the three parts above:

\[ Q_0 = Q_0(E) + Q_0(Q_M) + Q_0(Q_L) = 243.6 + 103.1 + 26.2 = 372.9 \text{ MWh} \ . \]

2) Allocate total fuel consumption by CHP plant products

The fuel consumption for electricity production is:

\[ B_E = B \cdot \frac{Q_0(E)}{Q_0} = 48.4 \cdot \frac{243.6}{372.9} = 31.6 \text{ tef} \ . \]

Analogously, the fuel consumption for industrial steam and heat production is:

\[ B_M = B \cdot \frac{Q_0(Q_M)}{Q_0} = 48.4 \cdot \frac{103.1}{372.9} = 13.4 \text{ tef} \ , \]
\[ B_L = B \cdot \frac{Q_0(Q_L)}{Q_0} = 48.4 \cdot \frac{26.2}{372.9} = 3.4 \text{ tef} \ . \]

3) Calculate fuel rates and product costs

Fuel rates for CHP plant products net supply are:

\[ b_E = \frac{B_E}{E - E^{AUX}} = \frac{31.6}{80 - 3} = 0.411 \text{ tef MWh} \ , \]
\[ b_M = \frac{B_M}{Q_M - Q_M^{AUX}} = \frac{13.4}{144.2 - 0} = 0.093 \text{ tef MWh} \ , \]
\[ b_L = \frac{B_L}{Q_L - Q_L^{AUX}} = \frac{3.4}{93 - 0} = 0.037 \text{ tef MWh} \ . \]

And product costs are:
\[ Z_E = b_E \cdot Z_B = 0.411 \cdot 100 = 41.1 \, \text{EUR MWh}, \]
\[ Z_M = b_M \cdot Z_B = 0.093 \cdot 100 = 9.3 \, \text{EUR MWh}, \]
\[ Z_L = b_L \cdot Z_B = 0.037 \cdot 100 = 3.7 \, \text{EUR MWh}. \]

The additional calculation is shown in the xlsx-appendix.

4. The methods comparison

Considering the fact that the thermodynamic and economic cost allocation methods have been developed to achieve different goals, the methods are compared separately by group.

4.1. Thermodynamic methods comparison

4.1.1. Comparison principles

The most important feature of the thermodynamic methods is their ability to take into consideration a) the differences between thermal (steam) energy and electric energy, b) the differences between various types of thermal (steam) energy. The comparison of the thermodynamic methods is performed in two stages.

In the first stage for each method, I examine whether it allows considering the differences mentioned above. If both kinds of differences are exhibited, I regard such a method as effective; if only one type of difference is present, the method is regarded as less effective; finally, if no differences are considered, the method is marked ineffective.

In the second stage, I order the methods within the corresponding subgroup as their applicability decreases (effective, less effective, and ineffective).

4.1.2. Efficiency, advantages, and disadvantages

Based on the principles stated in the previous section, I compared the thermodynamic methods. The results are given in Table. 2.

Table 2: Comparison of Thermodynamic Cost Allocation Methods

<table>
<thead>
<tr>
<th>No.</th>
<th>Method</th>
<th>Differences between</th>
<th>Differences between</th>
<th>Mechanisms to</th>
<th>Efficiency</th>
</tr>
</thead>
</table>

Mathematical bureau
1) The exergy method (sections 2.2.2, 2.3.1) is the most commonly used and considered the most "fair" and exact thermodynamic method [2, 4]. It allows examining the differences between various types of energy using exergy. Its main disadvantages are a large number of calculations and a significant number of input data [2, 4].

2) The work method (section 2.2.4) is one of the most commonly used thermodynamic methods since it allows examining the differences between various types of energy using the technical characteristics of the CHP plant equipment [2, 4]. The main disadvantages of this method are a large number of calculations and a significant number of input data [2, 4].

3) A new method proposed in this paper takes into consideration the differences between various types of energy using the coefficients of the linear steam turbine characteristic curve. Another advantage of this method is calculation simplicity. The disadvantage of this method is the necessity to find the linear steam turbine characteristic curve equation (section 3.2).

4) The advantage of the proportional distribution method by ORGRES (section 2.2.3) is the consideration of various types of thermal energy using special coefficients. Its disadvantage is the lack of comparison between electrical energy and thermal energy. The other disadvantages of this method are a large number of calculations, a significant number of additional input data, and a
confusing calculation sequence [4].

5) The steam parameters consideration method (section 2.2.5) is similar to the proportional distribution method by ORGRES in its advantages and disadvantages [4].

6) The “physical” method (section 2.2.1) is mostly criticized by researchers [3, 4, 8]. This method ignores the differences between the types of energies and does not meet the Second Law of Thermodynamics [4, 8]. According to the author [4], during USSR time, the background of the method application is the following: "The application of this method has allowed to artificially reduce the CHP plants electricity fuel rates and, without any technical development, take leading positions in the world energy industry." The use of this method as an official approach leads to artificial increases in heat tariffs [3]. The only advantage of this method is its simplicity [4].

4.1.3. Numeric analysis

In this paper, all the above-considered thermodynamic cost allocation methods have been implemented. The input data for the calculations are taken from the guidelines [4]. The results (fuel rates and product costs) are presented in the xlsx-appendix. As I mentioned in the Preface to the English version, the fuel rate values are being compared.

The obtained fuel rate values for the co-generated CHP plant products are gathered in Table 3 and illustrated in Fig. 1.

Table 3: Fuel Rate Values for CHP Plant Products Obtained with Thermodynamic Cost Allocation Methods

<table>
<thead>
<tr>
<th>No.</th>
<th>Method</th>
<th>Fuel Rate, tef/MWh</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Electricity</td>
</tr>
<tr>
<td>1</td>
<td>Exergy method (section 2.2.2)</td>
<td>0.331</td>
</tr>
<tr>
<td>2</td>
<td>Work method (section 2.2.4)</td>
<td>0.362</td>
</tr>
<tr>
<td>3</td>
<td>A new method on linear steam turbine characteristic curve (section 3)</td>
<td>0.411</td>
</tr>
<tr>
<td>4</td>
<td>Proportional distribution method by ORGRES (section 2.2.3)</td>
<td>0.320</td>
</tr>
<tr>
<td>5</td>
<td>Steam parameter consideration method (section 2.2.5)</td>
<td>0.425</td>
</tr>
<tr>
<td>6</td>
<td>“Physical” method (section 2.2.1)</td>
<td>0.218</td>
</tr>
</tbody>
</table>
For the methods being considered, the value $b_E$ is within the range of 0.218 to 0.425 tef/MWh; the value of $b_M$ is 0.090 to 0.134 tef/MWh; the value of $b_L$ is 0.030 to 0.133 tef/MWh. Therefore, depending on the method used, the fuel rate values for the products supplied differ two-fold.

Note that, the minimum electricity fuel rate value is obtained using the “physical” method (No. 6), and the maximum value is obtained using the steam parameters consideration method (No. 5). Both of these methods are the least effective thermodynamic methods (Table 2). Moreover, Fig. 1 shows that the “physical” method is the only thermodynamic method that does not allow taking into account the differences between the energies of industrial steam and heat.

The economic consequences of using the “physical” method to establish heat tariffs and the electricity prices should be emphasized. Firstly, the overestimated heat tariff allows CHP plants to operate ineffectively in the wholesale electricity market, shifting a part of the fuel consumption from electricity to heat, and, consequently, shifting a part of the cost. Secondly, the overestimated heat tariff reduces the competitiveness of the thermal energy produced by CHP plants in comparison with water-boiler houses [3]. Thus, the application of the “physical” method to solve the economic problems of establishing heat tariffs and bidding in the day-ahead and imbalance markets slows down the electricity and heat markets development.
4.2. Economic method comparison

4.2.1. Comparison principles

The most important feature of the economic cost allocation methods is the possibility to allocate co-generation benefits among the CHP plant products. Remember, the benefit is the reduction of the total fuel consumption and, as a result, the total cost in combined mode production compared to separate modes. The comparison of the economic methods is performed in two stages as well.

In the first stage, I investigate whether the methods allow allocating the benefit among the CHP plant products. If a method supports flexible benefit allocation depending on the market expectations, it is considered effective; if a method allocates benefit among the different products at a fixed ratio only, it is marked as less effective; finally, if a method attributes the benefit to only one of the CHP plant products, it is regarded as ineffective.

In the second stage, the effective methods are ordered within its subgroup according to the number of variables that allow benefit allocation: the more variable the method has the higher its ranking in the table below. And within the second subgroup of ineffective methods, the methods are ordered according to their applicability: the more popular the method is the higher its ranking in the table below.

4.2.2. Efficiency, advantages, and disadvantages

The comparison of the economic cost allocation methods is summarized in Table 4.

<table>
<thead>
<tr>
<th>No</th>
<th>Method</th>
<th>Benefit allocation among the CHP plant products</th>
<th>Benefit allocation mechanisms</th>
<th>Efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Benefit and risk-sharing method (section 2.3.7)</td>
<td>Allows flexible benefit allocation depending on the market expectations</td>
<td>Variable efficiency of water boiler, electricity market price risk variable</td>
<td>Effective</td>
</tr>
<tr>
<td>2</td>
<td>Method of an alternative way of heat supply (section 2.3.4)</td>
<td>Allows flexible benefit allocation depending on the market expectations</td>
<td>Variable efficiency of water boiler</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Benefit distribution method (section 2.3.6)</td>
<td>Allows only fixed ration benefit allocation</td>
<td>Consideration of both heat and electricity alternative ways of supply</td>
<td>Less effective</td>
</tr>
</tbody>
</table>
4. Method of an alternative way of electricity supply (section 2.3.5) does not allow, puts all benefit to heat supply. Ineffective

5. Energy method (section 2.3.3) does not allow, puts all benefit to electricity supply. Ineffective

1) The benefit and risk-sharing method (section 2.3.7) is the most flexible. Its advantage is the ability to consider the risk of electricity market price changes [2]. The method supports benefit allocation among the CHP plant products depending on the market expectations. The disadvantage of this method is the significant volatility of the electricity market prices caused by its widespread use. Such volatility is unacceptable in countries with underdeveloped electricity and heat markets [2]. For example, in the Russian electricity market, price volatility is avoided by several regulating procedures including the exchange control of bidding day-ahead prices.

2) The method of an alternative way of heat supply (section 2.3.4) is one of the most widely used cost allocation methods. The water-boiler efficiency taken as a variable allows flexible benefit allocation between heat and electricity, depending on market expectations [2]. This flexibility is an advantage of the method. The widespread use of this method leads to a higher volatility of the market prices, which can be regarded as both its advantage and disadvantage depending on the considered country.

3) The benefit distribution method (section 2.3.6) is also widely used in countries with developed electricity and heat markets. The disadvantage of this method is the lack of flexibility in benefit allocation; the benefit is distributed between electricity and heat in accordance with a fixed ratio. Still, this distribution mechanism is considered as an advantage of this method [2].

4) The method of an alternative way of electricity supply (section 2.3.5) is less widely used in practice. Its disadvantage is benefit allocation for heat production, which reduces the CHP plant competitiveness in the electricity market. The advantage of this method is its simplicity [2].

5) The energy method (section 2.3.3) is the least flexible economic cost allocation method. Its only advantage is the simplicity of calculations [2, 6].

4.2.3. Numerical analysis

All of the above-considered economic cost allocation methods have been implemented in this paper. The input data for the calculations are taken the same way as for the thermodynamic
methods (section 4.1.3). The results are presented in Table 5 and Fig. 2. Remember, numerical analysis is performed on the fuel rate values.

Table 5: Fuel Rate Values for CHP Plant Products Obtained with Economic Cost Allocation Methods

<table>
<thead>
<tr>
<th>No.</th>
<th>Method</th>
<th>Fuel Rate, tef/MWh</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Electricity</td>
</tr>
<tr>
<td>1</td>
<td>Benefit and risk-sharing method (section 2.3.7)</td>
<td>0.241</td>
</tr>
<tr>
<td>2</td>
<td>Method of an alternative way of heat supply (section 2.3.4)</td>
<td>0.219</td>
</tr>
<tr>
<td>3</td>
<td>Benefit distribution method (section 2.3.6)</td>
<td>0.293</td>
</tr>
<tr>
<td>4</td>
<td>Method of an alternative way of electricity supply (section 2.3.5)</td>
<td>0.422</td>
</tr>
<tr>
<td>5</td>
<td>Energy method (section 2.3.3)</td>
<td>0.159</td>
</tr>
</tbody>
</table>

In the method of alternative heat supply, the hot-water boiler efficiency is taken equal to 93% (section 2.3.4). In the benefit and risk-sharing method, the expected change of the electricity market prices is taken \( r = 0.1 \) (section 2.3.7).
For the economic cost allocation methods, the corresponding ranges of the specific consumption of equivalent fuel have the following values: $b_E = 0.159$ to $0.422$ tef/MWh; $b_M = 0.067$ to $0.153$ tef/MWh; $b_L = 0.067$ to $0.153$ tef/MWh. It should be pointed out that the economic methods do not take into account the differences between industrial steam and heat energies: the values of $b_M$ and $b_L$ are the same for most of the methods.

**Conclusion**

The following goals have been achieved in this work:

1) A review has been provided for the co-generated CHP plant products cost allocation methods most often used in Russia, as well as in countries with developed electricity and heat markets. The methods are divided into two groups: thermodynamic and economic.

2) A new thermodynamic cost allocation method has been introduced based on the linear characteristic curve of a steam turbine. This method takes into account the differences between the energy quality of the CHP plant products. The developed method eliminates one of the disadvantages common for effective thermodynamic methods: it simplifies calculations and reduces the scope of input data.

3) The thermodynamic and economic methods have been compared on the basis of the formulated principles and numerical results. The exergy method and the work method have been proved to be the most effective thermodynamic methods. Their main disadvantages are a significant number of input data and computational complexity. The benefit and risk-sharing method and the method of alternative heat supply have turned out to be the most effective economic methods. For certain countries, their main disadvantage is a high volatility of the market prices caused by the widespread use of these methods.

4) It is clearly shown that the use of the least effective thermodynamic method, the “physical” method, for assessing the CHP plant product cost slows down the electricity and heat market development.

Promising areas for further research are:

1) To develop a set of formal effectiveness criteria for each group of the methods; to assess the selected methods using these criteria and formulate official recommendations.

2) To revise the applicability of the methods. It is advisable to use the thermodynamic methods for internal quality assessment of the CHP plant operation modes. The economic methods
are recommendable for establishing the heat tariffs and bidding in the wholesale electricity market.

References

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